THE GEOMETRY OF HOLOGRAPHY

(OR WHAT CAN GEOMETRY CAN TEACH US ABOUT HOLOGRAPHY?)

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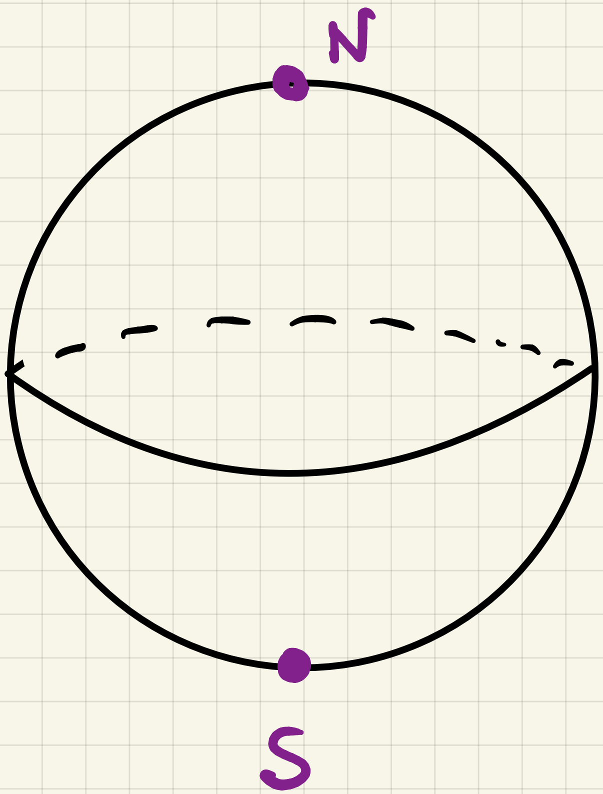
FISMAT DAYS 2024

BASED ON: Phys. Rev. D 108, 126005 (2023)
+ ONGOING (UNPUBLISHED) WORK

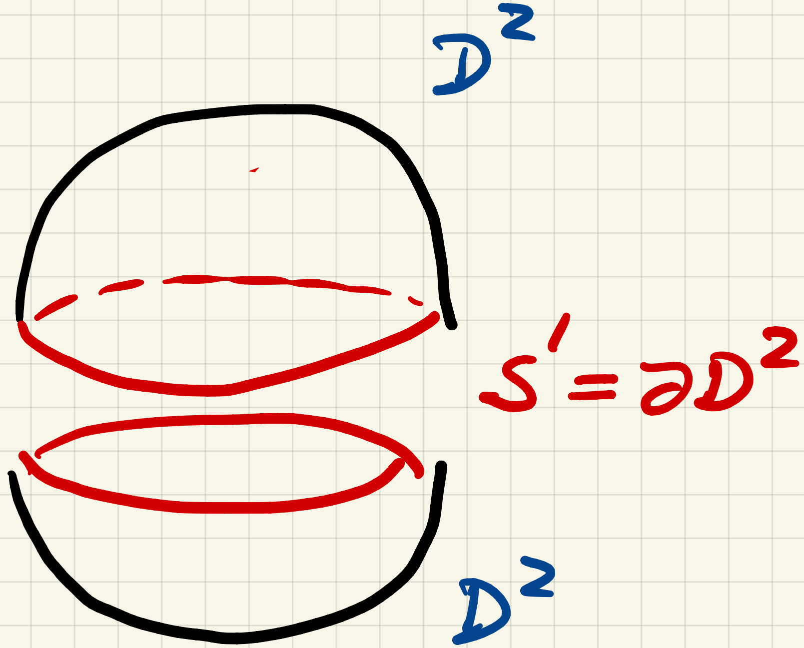
MOTIVATED BY JOINT WORK WITH R. GOVER, R. OLEA, P. SUNDELL, & A. WALDRON

MOTIVATION

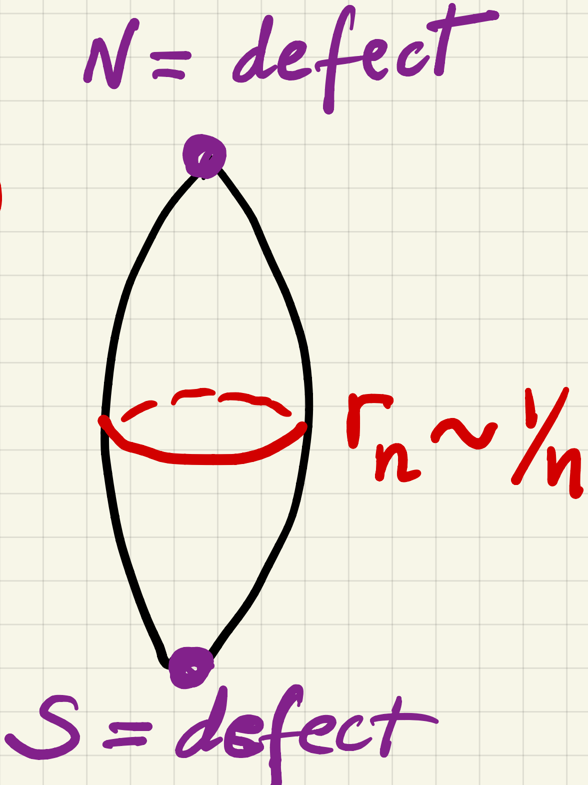
$S^2 \hookrightarrow \mathbb{R}^3$



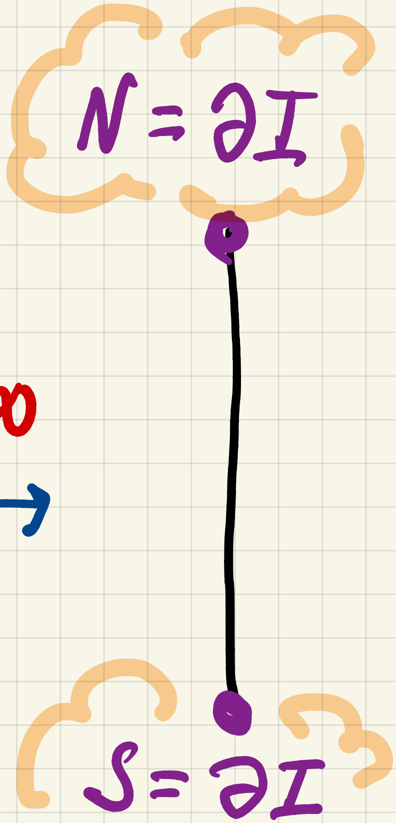
CUT



$S^2/\mathbb{Z}_n$



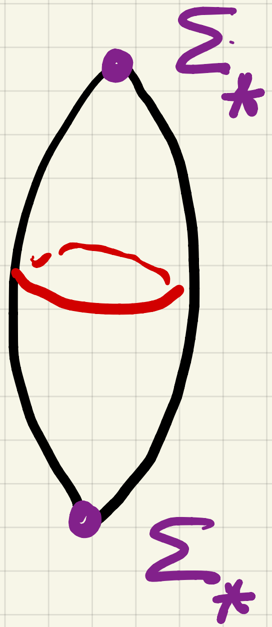
$n \to \infty$



$$\{N, S\} = \mathcal{F}(S^2/\mathbb{Z}_n)$$

IDEA : STUDY

$$\boxed{X^{d+2} \cong S^2/\mathbb{Z}_n \times \Sigma^d}$$



$$\Sigma_* = \Sigma\Big|_{N,S} \quad (\Sigma \text{ has no boundary})$$

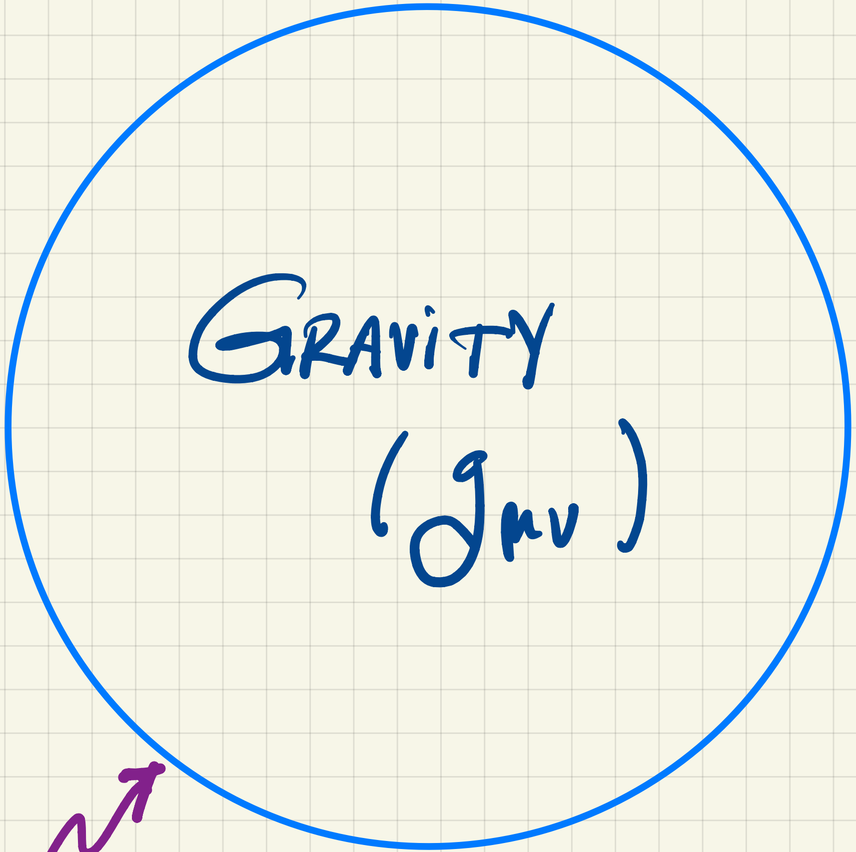
CLAIM : DEFECT-TO-BOUNDARY TRANSITION

$$\Sigma_*^d \hookrightarrow X^{d+2} \xrightarrow{n\to\infty} \Sigma^d = \partial M^{d+1}$$

$$(M^{d+1} \cong I \times \Sigma^d)$$

IS THIS USEFUL?

GRAVITY ON AdS^{d+1}



BOUNDARY CFT

$$\mathcal{H}_{GRAV} \cong \mathcal{H}_{CFT}$$

$$\ell_s \ll L_{AdS}$$

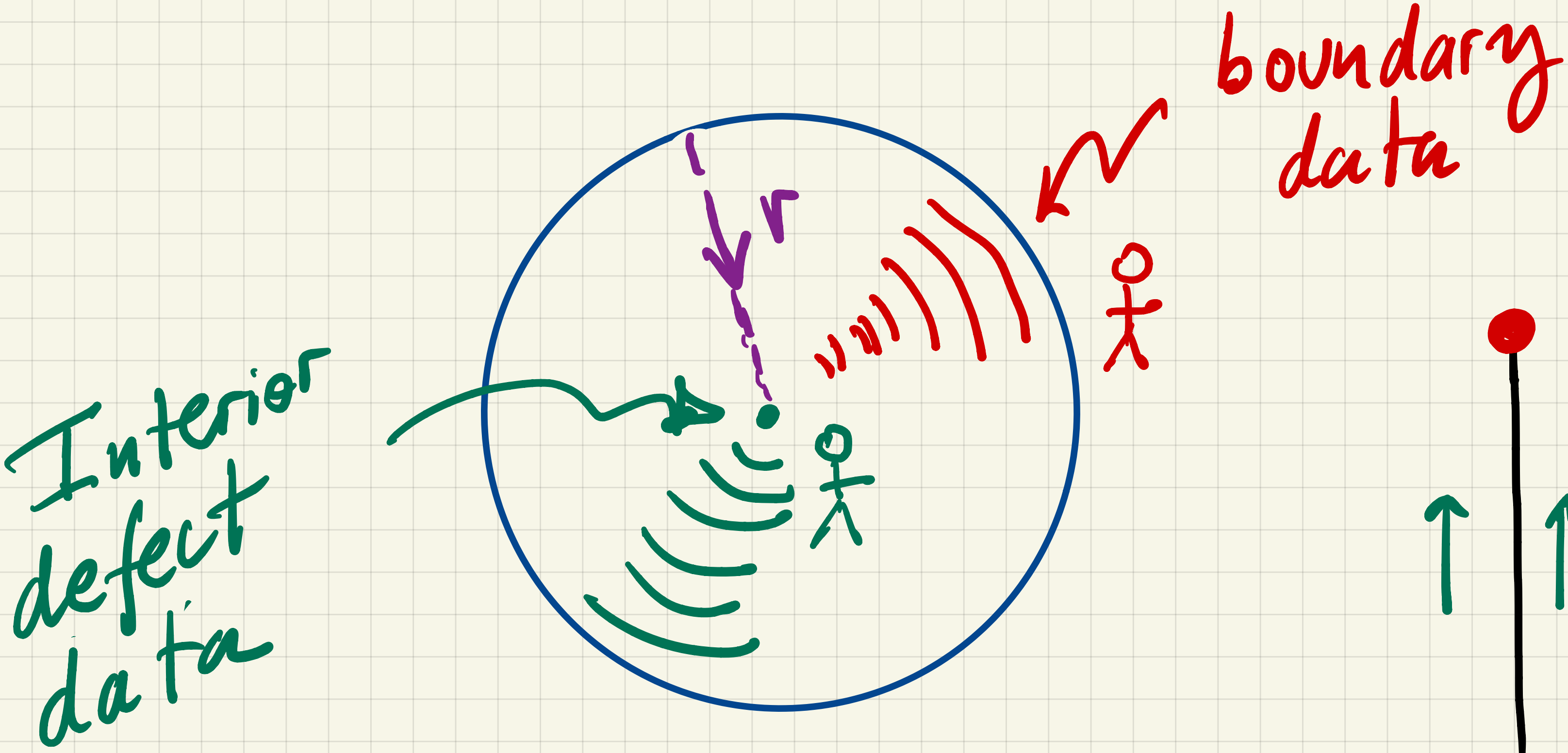
CLASSICAL LIMIT

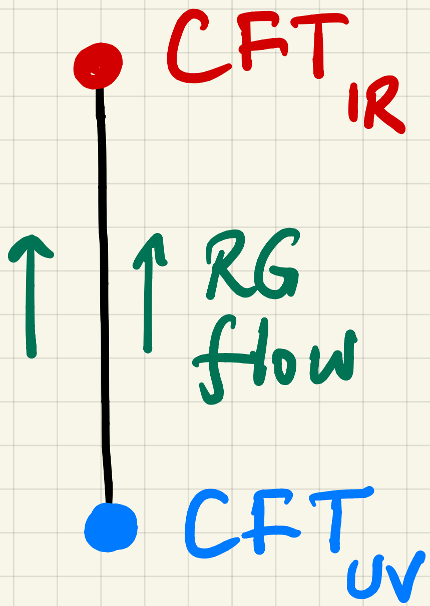
$$Z_{CFT} = Z_{GRAV} \approx e^{I[g]}$$

EINSTEIN

$$\langle T(z)\, T(\bar{z}) \rangle \sim \frac{c}{(z-\bar{z})^4}$$

* AdS / CFT $\Rightarrow$ $\boxed{\text{SPACETIME RECONSTRUCTION}}$





* Conjecture 1 : RECONSTRUCTION FROM DEFECT DATA

* Conjecture 2 : DUAL GAUGE THEORIES ALSO LIVE ON DEFECTS

* Conjecture 3 :

$$
\begin{tikzcd}
\Sigma^* \arrow[r, hook] & X^{d+2} \arrow[r, "n\to\infty"] & \Sigma = \partial M^{d+1} \\
\mathrm{CFT}_{IR} \arrow[u, squiggly] \arrow[rr] & & \mathrm{CFT}_{UV} \arrow[u, squiggly]
\end{tikzcd}
$$

OUTLINE

1. LOCAL GEOMETRY OF THE AdS BOUNDARY
2. AdS WITH DEFECTS & LOCAL DEFECT GEOMETRY
3. BOUNDARY RECONSTRUCTION ($n \to \infty$) & HOLOGRAPHY
4. GENERAL RULES

1. THE BOUNDARY OF AdS^{d+1}

THEOREM (GRHAM-LEE '91)

LET (M^{d+1}, g) BE ASYMPTOTICALLY AdS.

THEN

$$\boxed{g = \frac{dr^2 + h(r)}{r^2}}$$

ON $\Sigma \times [0, \varepsilon)$.

THEOREM (FEFFERMAN–GRAHAM '85)

THE EINSTEIN CONDITION $R_{ij}(g) + d\,g = 0$
CAN PERTURBATIVELY
SOLVED AS

boundary data

$$h = h_{(0)} + r^2 h_{(2)} + \dots\dots$$

WHERE

$$h_{(2)\,ij} = -P^{\Sigma}_{ij} \qquad (d \neq 2)$$

$$\boxed{\mathrm{Tr}\left(h^{-1}_{(0)} h_{(2)}\right) = -\frac{1}{2} R^{\Sigma}} \qquad (d = 2)$$

THE FG-EXPANSION AMOUNTS TO COMPUTING THE BOUNDARY WEYL ANOMALY IN AN HOLOGRAPHIC FASHION (Henningson-Skenderis '98)

$$S \underset{d=2}{=} \frac{1}{16\pi G_3} \int_{M3} dr\, d^2x \sqrt{|\det g_M|}\,(R-2\Lambda) + \dots$$

$$\approx \frac{1}{4\pi G_3} \int_{\Sigma}\int_{r>\varepsilon} \frac{dr}{r^3}\left(1+\frac{r^2}{2}\underbrace{\mathrm{Tr}(h_{(0)}^{-1} h_{(2)})}_{\text{Known (FG)}} + \dots\right) dV_\Sigma$$

$$\Rightarrow \boxed{S = \frac{a}{\varepsilon^2} + A \log \varepsilon + \text{finite terms}}$$

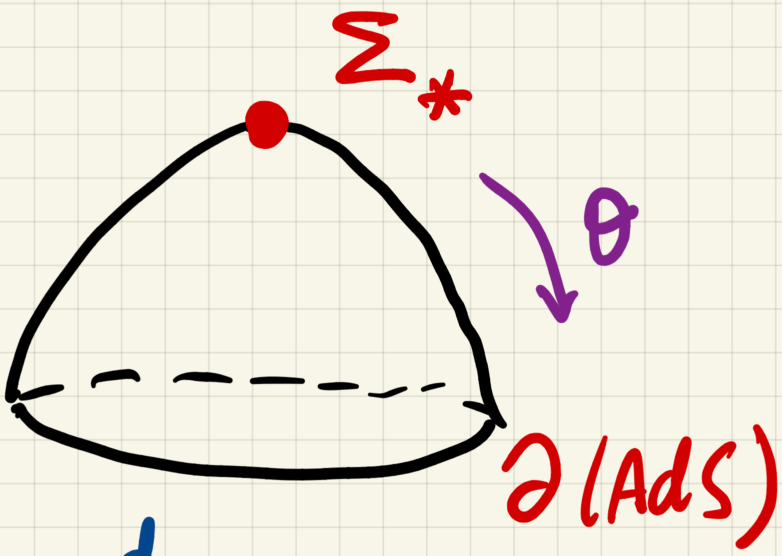
$$a = \frac{\mathrm{Vol}(\Sigma)}{8\pi G_3} \qquad A = \int_\Sigma \mathcal{A} = \frac{\chi(\Sigma)}{4G_3}$$

WHERE

BROWN-HEANNEAUX

$$\mathcal{A} = \frac{\ell}{16\pi G_3} R^\Sigma \overset{!}{=} \frac{c}{24\pi} R^\Sigma \Rightarrow \boxed{c = \frac{3\ell}{2G_3}}$$

2. AdS with defects



LEMMA :

GLOBALLY

1) $AdS^{d+2} \cong D^2/\mathbb{Z}_n \times \Sigma^d$

2) $$g_{AdS} = \frac{R_0^2\left(d\theta^2 + \sin^2\theta \, \frac{d\phi^2}{n^2}\right) + h(\theta, x)}{u^2(\theta)}$$

WHERE

$$u = \cos\theta \; ; \quad R_{ij}(h) + (d-1)h = 0$$

THEOREM (C. ARIAS '23)

LOCALLY ABOUT Σ_*

$$u = u_0 + \theta^2 u_2$$

$$h_{ij} = h_{(0)ij} + \theta^2 h_{(2)ij} + \cdots$$

WHERE

$$h_{(2)ij} = \frac{R_0^2}{2} R_{ij}^{\Sigma_*} + \left(2\frac{u_2}{u_1} + \frac{(d+1)R_0^2}{2L^2 u_0^2} \right) h_{(0)ij}$$

$$R_{\Sigma_*} = \frac{2}{R_0^2}\left(1 + 4\frac{u_2}{u_0} + \left(1 - \frac{d}{2}\right)\frac{(d+1)R_0^2}{L^2 u_0^2} \right)$$

LEMMA (DEFECT CENTRAL CHARGE)

$$T^{\Sigma_*}_{ij} = \frac{1}{4G_4}\left(1-\frac{1}{n}\right) h_{(0)ij}$$

WHEN Σ_* IS A 2-MANIFOLD

$$\mathrm{Tr}\left(T^{\Sigma_*}\right) = \frac{1}{2G_4}\left(1-\frac{1}{n}\right) \stackrel{!}{=} \frac{c_*}{24\pi} R_{\Sigma_*}$$

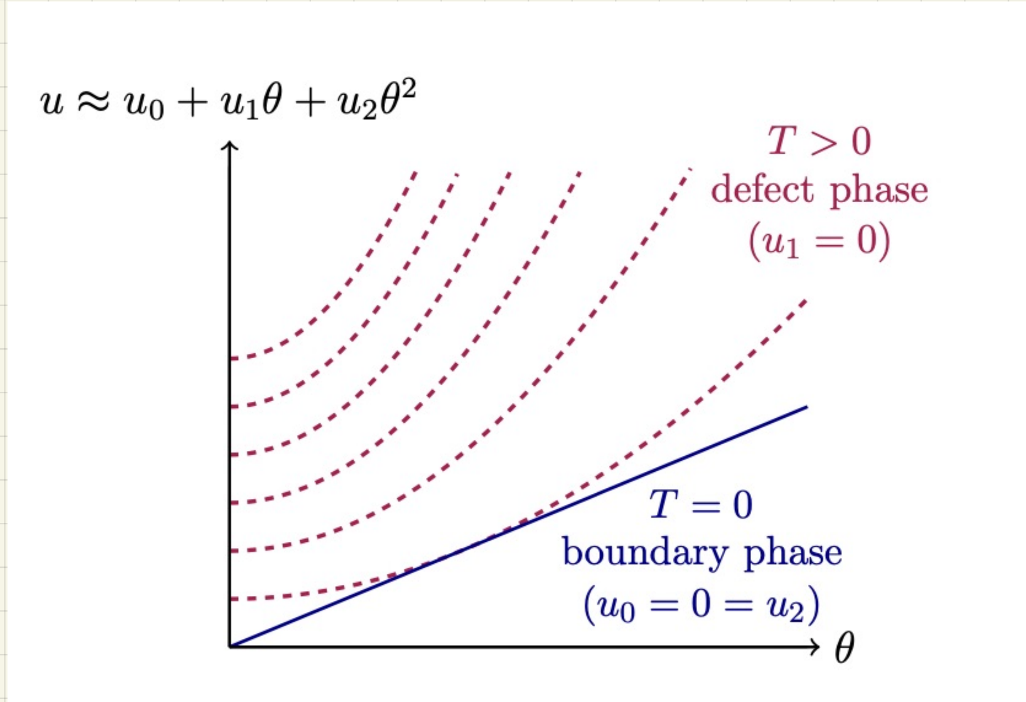
$$\Rightarrow \boxed{c_* = \left(1-\frac{1}{n}\right)\frac{3R_0}{(1+4\mu)G_3}}$$

3. BOUNDARY RECONSTRUCTION $(n \to \infty)$

CONSISTENCY OF $\Sigma_* \to \Sigma$ LIMIT REQUIRES

1 $$u = \begin{cases} u_0 + \theta^2 u_2 & (n \text{ finite}) \\ \theta u_1 & (n \to \infty) \end{cases}$$

ORDER PARAMETER!

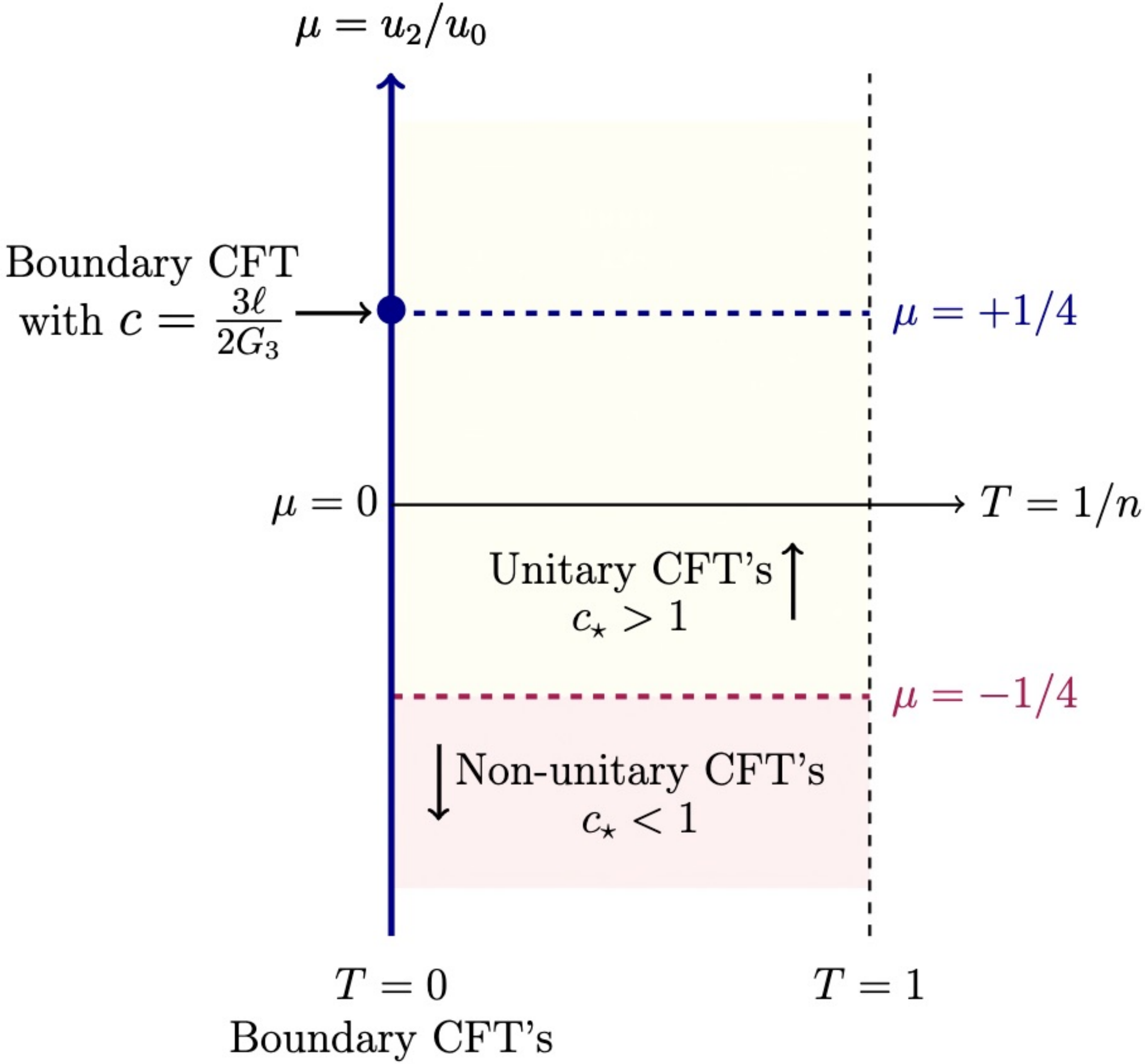


② $R_0 \longrightarrow \ell$ (AdS^3 radius)

THE ABOVE IMPLIES

$$C_* \longrightarrow \boxed{C = \frac{3}{(1+4\mu)} \frac{\ell}{G_3}}$$

ONE-PARAMETER FAMILY OF CENTRAL CHARGES ($\mu := u_2/u_0$)

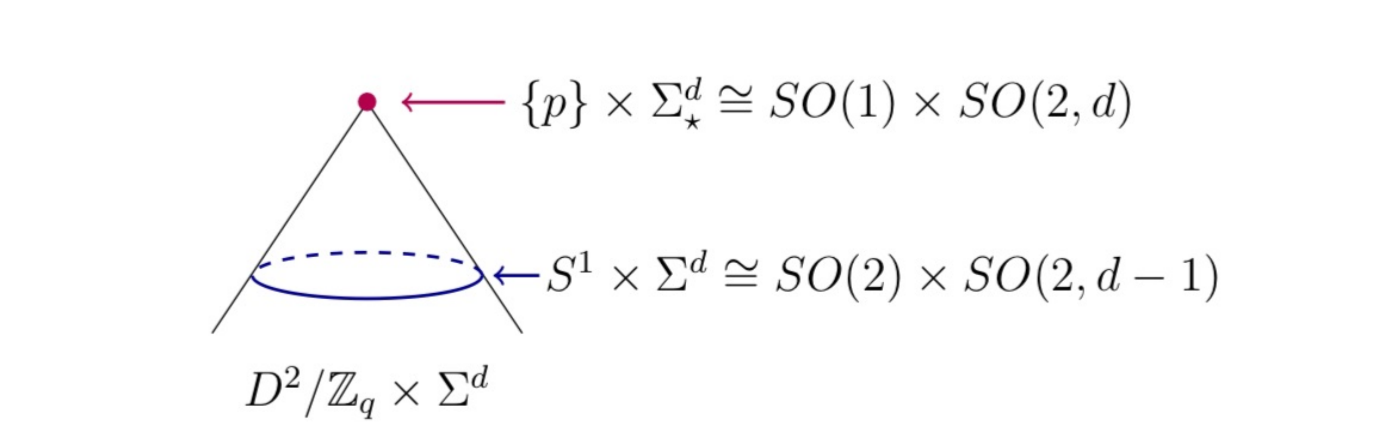


$\mu = u_2/u_0$
Boundary CFT
with $c = \frac{3\ell}{2G_3}$
$\mu = +1/4$
$\mu = 0$
$T = 1/n$
Unitary CFT's
$c_\star > 1$
$\mu = -1/4$
Non-unitary CFT's
$c_\star < 1$
$T = 0$
Boundary CFT's
$T = 1$

4. GENERAL RULES

ON GENERAL GROUNDS, ANY CONCRETE REALIZATION OF THE HOLOGRAPHIC PRINCIPLE IN GRAVITY MUST OBEY A SET OF GEOMETRIC RULES, REGARDLESS OF THE DETAILS OF THE THEORY.

1. SYMMETRIES



Recall that $SO(2,d+1) \supset SO(p) \times SO(2,q)$

$$p+q = d+1$$

2. Existence of $T_{\mu\nu}$

$$\Sigma_*^d \xrightarrow{\delta^2} X^{d+2} \Rightarrow \boxed{T \sim \left(1-\frac{1}{n}\right) h^{(0)}}$$

3. Conformal classes of metrics

4. Well defined asymptotics

CONCLUSIONS

1) THERE EXISTS A $\Sigma_* \to \Sigma$ TRANSITION WHEREBY THE AdS BOUNDARY ARISES FROM A DEFECT SUBMANIFOLD

2) HOLOGRAPHIC/DUAL THEORIES DO NOT NECESSARILY HAVE SUPPORT ON BOUNDARY SUBMANIFOLDS.

3) DUAL CFT ON $\partial(\text{AdS})$ IS THE $n \to \infty$ OF A MORE FUNDAMENTAL THEORY.

4) 2 DIFFERENT TYPE OF FLOWS