Unfolded Quantum Fields

Per Sundell

Instituto de Ciencias Exactas y Naturales
Universidad Arturo Prat
Iquique

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Outline

Introduction

Goal

Unified framework for quantum mechanics (QM), gauge theory and gravity in which

- quantum states collapse dynamically
- novel representations of noncompact symmetry groups arise as boundary conditions on fields (modifying ultra-violet and infra-red limits of particle physics and cosmology)

Two connected ideas from string theory

- Nonlinear QM with state-operator correspondence yields moduli spaces of noncommutative (NC) geometries ..... identifiable as....
- Classical solution spaces of topological quantum field theories (TFT) whose operator algebras contain spacetimes with quantum fields (QFT)

Progress on first-quantized particles

- Classical solution spaces to Vasiliev's higher-spin gravities (HSG) $\rightsquigarrow$ holomorphic, metaplectic representations of the complexified symplectic group
  - Double covering from square-root prefectors
  - Unitary evolution operators in the real interior
  - Twisted boundary conditions in the complex interior
  - Group-algebra projectors at asymptotic boundaries
- Classical solution spaces to integrable extensions of Einstein's theory of gravity (GR) including dark fluids $\rightsquigarrow$ extensions of Dirac's conformal particle
  - SpH(2)-gauged $3n$-dimensional Poisson manifolds containing Sp(2)-gauged $2n$-dimensional symplectic leaves
  - Nonlinear evolution along proper Poisson directions

Hypothesis for QM

- Nonlinear QM contains projective evolution operators reaching projectors in finite time
- Deterministic albeit chaotic dynamics in which quantum states collapse at defects

Progress on second-quantized fields

Deformation quantization of HSG as NC TFT

- Quantum fields emerge as cohomology elements inside boundary operator algebras
- Resolution of curvature singularities of Petrov Type D, e.g., Coulomb and Schwarzschild singularities
- Side-steps thorny non-locality issues of the Fronsdal program for computing matrix elements of time-evolution operators by summing field histories on spacetime backgrounds
- On-shell actions of NC TFT encode correlation functions of holographically dual conformal field theories (CFT)

Hypothesis for QFT

- Holographic dualities encoded into vacuum states of second-quantized NC TFTs on open manifolds with entangled boundary operator algebras
- Entangled vacua given by operators in direct-product algebras obeying overlap conditions formulated using isomorphic subalgebras of observables (Morita equivalence?)

Interplay between first- and second-quantized systems

- On-shell actions of NC TFT identifiable as generating functional of vertex operator algebras of first-quantized systems exhibiting state-operator correspondence, e.g., conformal particles.
- Deformed BRST operators of conformal particles including twisted sectors and branes form classical solution space of fractional-spin gravity (FSG)
- Vasiliev's HSG and CFT models coupled to conformal HSG arise as distinct classically consistent truncations of FSG

Impact on first-quantized models

- Fronsdal on-shell curvatures, e.g., scalar fields, Faraday tensors and Weyl tensors, mapped to pseudo-real Weyl zero-form Ψ of HSG
- Linearized HSG equation of motion identified as Heisenberg-Liouville equation of first-quantized system with density matrix $\Psi\Psi^\dagger$, c.f., Koopman-von Neumann classical quantum mechanics
- Nonlinear corrections describe how the evolution of Ψ on symplectic manifold back-reacts to symplectic structure and Hamiltonian structure
- Resulting nonlinear QM obeys the variational principle with action given by that of FSG

Unified TFT framework

Employ the Alexandrov-Kontsevich-Schwarz-Zaboronski (AKSZ) solution to the Batalin-Vilkovisky (BV) master equation

- Classical solutions are Q-morphisms
- C/NC Q-manifolds encode L_∞ / A_∞ algebras
- Cohomological equations of motion in the image of BV differential built using Cartan's exterior calculus
- Spacetime manifolds arise as integration constants of zero-forms valued in cosets of noncompact groups
- Local degrees of freedom arise as integration constants of zero-forms valued in modules of noncompact groups
- Partition functions are sums over topological manifolds of different dimensions weighted by universal master action

Cartan-Vasiliev unfolding and AKSZ quantization

- Partial differential equations of motion formulated as universally Cartan integrable systems (CIS) of constraints on differential forms
- Integration subject to boundary conditions yields classical solution spaces given by gauged, graded Lie groupoids alias Cartan integration modules (CIM)
- Partition functions computed using universal topological sigma models given by sums over differential forms on open manifolds of different dimensions
- Boundary conditions given by restrictions of CIMs
- Boundary operator algebras represented by symbols given by classical observables belonging to BV cohomologies

Rest of talk

Outline how quantum fields with holographic properties emerge within boundary operator algebras of (NC) AKSZ sigma models with classical moduli spaces given by describing gauged, graded Lie groupoids with infinite-dimensional zero-form modules

Ordinary differential equations

ODE of order n contained in system of $n+1$ ODEs of order 1, viz.,

$$dx^\mu \approx e\,K^\mu(x;t)\ , \qquad de \approx 0\ , \qquad dt \approx e\ ,$$

where $(x^\mu, e; t)$, $\mu = 1, \ldots, n$, are forms of degrees $(0, 1; 0)$, respectively.

Let $y^M = (x^\mu; t)$ and $\vec{V} = V^M(y)\vec{\partial}_M$ with $V^M = (K^\mu; 1)$. Locally defined solutions are given by

$$y^M \approx e^{\lambda'\vec{V}'} y^M\ , \qquad e \approx d\lambda' + e'\ ,$$

where λ' is a zero-form, referred to as the gauge function, and (y'^M, e') is a reference solution.

In particular, setting $e' = 0$ yields $dy_0'^M = 0$, i.e., $y_0'^M$ are integration constants.

Gauged abelian Lie groupoid

The classical solution space is a Lie groupoid

$$G \times E \overset{\hat{s},\hat{t}}{\Longrightarrow} E \ , \qquad \hat{s}(g,\psi) = \psi \ , \qquad \hat{t}(g,\psi) = g\psi \ ,$$

$$(g,\psi)(g',\psi') = (gg',\psi') \qquad \text{if} \quad \psi = g'\psi' \ ,$$

itself a representation of a Lie groupoid $G \overset{s,t}{\Longrightarrow} \check{G}$ on E in unison with natural bundle structure

$$G \times E \overset{\hat{\pi}}{\to} \check{G} \overset{\pi}{\leftarrow} E \ , \qquad \hat{\pi}(g,\psi) = s(g) = \pi(\psi) \ ,$$

$$\pi(\hat{t}(g',\psi')) = \pi(\hat{s}(g,\psi)) = \pi(\psi) = s(g) = t(g')$$

Decomposing

$$x^\mu \approx e^{\lambda' \vec{V}'} x'^\mu \ , \quad t = t' + \lambda' \ , \quad e \approx e' + d\lambda' \ ,$$

we identify

$$g := \lambda \ , \quad \psi = (x, e; t) \ , \quad gg' = \lambda + \lambda' \ , \quad \pi(\psi) = t \ .$$

Restriction to Dom(mult)

Let $X^\alpha := (x^\mu, e)$ coordinatize fibre of $L \hookrightarrow E \stackrel{\pi}{\to} \check{G}$. Assume $[\vec{\partial}_t, \vec{V}] = 0$. Then $\vec{Q} := Q^\alpha \vec{\partial}_\alpha$ with $Q^\alpha = (eK^\mu, 0)$ is a Q-structure on L, and X is a universally flat superconnection, viz.,

$$dX^\alpha \approx Q^\alpha(X) \ , \qquad Q^\beta \partial_\beta Q^\alpha \equiv 0 \ ,$$

a universally CIS. Let $\vec{T}_t := (dt^\alpha + t^\beta \partial_\beta Q^\alpha)\vec{\partial}_\alpha$, $t^\alpha = (0, t)$, be the generator of Cartan gauge transformations. The classical solution space is obtained by Cartan integration, viz.,

$$X^\alpha \approx e^{\vec{T}'_t} X'^\alpha \ , \qquad dX'^\alpha \approx Q^\alpha(X') \ ,$$

i.e., it is the domain of the multiplication map of

$$G \times L \Rightarrow L \ .$$

Gauged Lie grouped as CIS

Change perspective: Start from extended Cartan integrable system with forms $(X^\alpha, t^\alpha, X'^\alpha)$, curvature constraints

$$dX^\alpha \approx Q^\alpha(X) , \qquad X^\alpha \approx e^{\vec{T}'_t} X'^\alpha , \qquad dX'^\alpha \approx Q^\alpha(X') ,$$

and integration constants $(x_0'^\mu, t_0)$.

Germ of quantum field in the algebra of operators generated by sigma model.

Completion

- $x_0'^\mu \rightsquigarrow$ NC (graded) coordinates of (differential) Poisson manifold M
- Evolution along $t \in \mathbb{R} \rightsquigarrow$ noncompact, nonabelian spacetime symmetry group G
- $M \rightsquigarrow \infty$-dimensional G-module $\supset$ representations that are i) unitary; and ii) consist of tensors of structure subgroup $H \subset G$

Nonabelian gauged Lie groupoids

Let LM be the total space of a Lie algebroid with anchor map $\vec{\rho} : \Gamma(LM, M) \to \Gamma(TM, M)$. Let K_a be a basis for LM and $\vec{K}_a := \vec{\rho}(K_a)$, viz.,

$$[K_a, K_b]_{LM} = f_{ab}^c K_c \ , \qquad [\vec{K}^a, \vec{K}^a]_{TM} = f_{ab}^c \vec{K}_c \ .$$

Let x^μ and e^a, respectively, coordinatize the base and fibre of $L[1]M$. Universally Cartan integrable system

$$dx^\mu \approx e^a \, K_a^\mu(x) \ , \qquad de^a \approx \frac{1}{2} f_{bc}^a(x) e^b \wedge e^c \ .$$

Let $X^\alpha = (x^\mu, e^a)$, $Q^\alpha = (e^a \, K_a^\mu(x), \frac{1}{2} f_{bc}^a(x) e^b \wedge e^c)$, $t^\alpha = (0, t^a)$. The diagonal gauged Lie groupoid

$$dX^\alpha \approx Q^\alpha(X) \ , \qquad X^\alpha \approx e^{\vec{V}'_t} X'^\alpha \ , \qquad dX'^\alpha \approx Q^\alpha(X') \ ,$$

is an extended CIS with integration constants $(x_0'^\mu, t_0^a)$.

PDE generalization

Local degrees of freedom

- Take M to be ∞-dimensional G-module
- Classical perturbation theory $\rightsquigarrow$ Weyl zero-form module M comprising i) unitary representations U of G comprising observable quantum states; and ii) tensorial representation T of Lie algebra of H consisting of curvature tensors and their derivatives
- G does not act faithfully on T $\rightsquigarrow$ classical curvature singularities resolved in U

Graded geometry

- Extend $L[1]M$ to $\mathbb{Z}$-graded, homotopy L_∞-algebra
- Coordinate of degree $n \rightsquigarrow$ sum over forms of degrees p with with ghost numbers $n-p$ alias AKSZ superform
- The image of $\vec{Q}$ in degree zero consists of zero-form constraints selecting the shell of the system

Batalin-Vilkovisky path integral

Rewrite integrals over large numbers of real variables as superintegral using Berezin measures drawn from the superdeterminant

$$\mathrm{Ber}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det(A - BD^{-1}C)\det(D^{-1})$$

QPS structure

- Start from integral $\oint_S \omega$ of de Rham closed form ω over orientable submanifold S of real manifold R
- Insert δ_C-form and convert to Berezin integral over $T[1]R$
- Introduce measure μ_R and perform fermionic Fourier transform to Berezin integral over Lagrangian submanifold of graded symplectic $T^*[-1]R$
- de Rham differential converted to BV Laplacian
- Stationary phase obeys BV master equation

Alexandrov-Kontsevich-Schwarz-Zaboronsky's

- Configuration space consists of maps
$$\varphi : X \to Y$$
where X and Y are graded manifolds
- QPS-structure induced from graded Hamiltonian structure on Y and Q-structure and compatible Berezinian measure on X
- Natural extension to NC X and Y $\rightsquigarrow$ Frobenius-Chern-Simons model

Partial actions

BV cohomology elements K give by boundary functionals induced from differential forms on the target in strictly positive degree

- $\delta K \approx 0$ modulo Cartan curvatures
- Deformed vacua $\mathrm{Tr}_{\mathcal{B}} e^{i\mu K}$
- Unfolded GR coupled to matter has partial action given by Einstein-Cartan action plus first-order matter actions

Conclusion and Outlook

AKSZ quantization of FCS model for FSG on two-dimensional cylinder

- Massless particle states arise in U sector of HSG
- Fronsdal quantum fields arise in T sector of representations of locally defined spacetimes in G/H
- Idem unfolded CFT side

Work in progress

Construction of vacuum state entangling the U sectors of unfolded HSG and CFT